Actergy as a Reflex Performance Metric: Integral-Kinematics Applications

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***Abstract*—Actergy, the time-integral of energy, is employed in a human kinetic sensing system. This paper presents hardware and signal-processing mathematics to measure actergy in a health science, training or gaming environment, as a more detailed metric of performance than reflex time-delay. Fundamental quantities of Integral-Kinematics are compared alongside actergy, with further potential applications in aerospace.**

I. Introduction

We introduce a new type of physical fitness metric for training, human health science, and interactive gaming.

We propose *actergy*, the time-integral of energy expenditure (unlike power, the time-derivative of energy), as a new metric for reflex tests and reflex-based interactive games. Rather than previous reflex-measurement methods which use time-delay between stimulus and response as the metric for reflex performance [1]–[3], actergy takes into account the complete waveform of energy expenditure, not merely two data points (start time and end time). Actergy, as the time-integral of energy, is biased towards rewarding early power expenditure, making it well suited for continuously-varying reflex measurements.

By processing a continuously-varying signal, actergy accounts for different users who may have the same overall reflex time-delay, but different waveforms of energy reaction from one millisecond to the next. For example, one user may have zero reaction at first followed by a large reaction, while another user may have a weak, broadened curve of power expenditure.

Physical fitness training often includes: (1) Training over short bursts for maximum power, as measured in watts; (2) Endurance training over longer periods for maximum energy, as measured in joules. We propose a third class of physical fitness training, to maximize actergy, creating three types of energetic fitness:
(1) Power as measured in watts;
(2) Energy as measured in watt·seconds or joules;
(3) Actergy as measured in watt·seconds^2 or joule·seconds.
Each of these is the time-integral of the one before it. A physical fitness regimen could focus on all three: Power, Energy, and Actergy, to enhance different types of muscle and metabolic activity in short, medium and long segments of time.

Our actergy measurement system, which consists of hardware and signal-processing software, therefore is designed both as a metric for reflex tests and as a platform for rapid-action gaming to enhance physical fitness.

II. Transforming Weigh-Scale Signals to Actergy

We used a gas-cylinder weigh scale as an interface device (Fig. 3). The scale's output signal was connected to a National Instruments analog-to-digital converter, for signal processing on a computer where we converted the time-varying "mass" measurement into actergy. While actergy is the integral of energy, the conversion to actergy on our system is more involved than a simple integration. We derive the actergy conversion as follows.

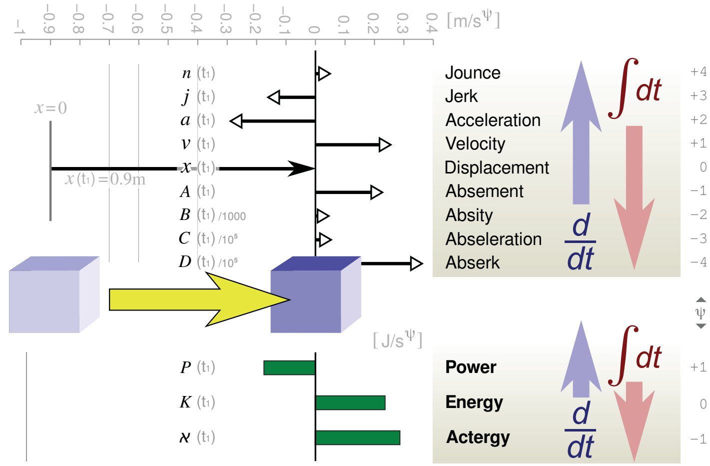

Fig. 1. The time-integrals of displacement are fundamental kinematic quantities that complete the sequence of "kines" of displacement in both directions. Energy can also be time-integrated, leading to actergy. Actergy we propose as a generalized analogue to *Lagrangian action*, which itself is specifically an integral of kinetic energy minus potential energy of a system.

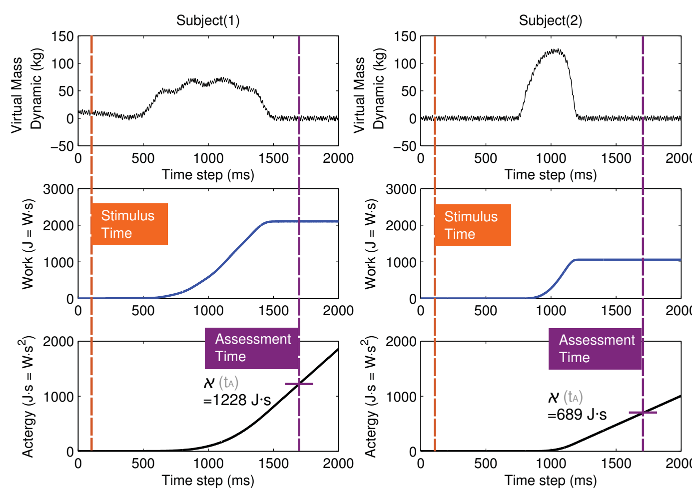

Fig. 2. Reflex test data from gas-cylinder scale, converted to actergy for two subjects, using the conversion formulas derived in this paper. Actergy is a metric that favours early energy expenditure, while it also considers all energy data points in a range rather than merely a time delay measurement.

We took an initial sample of DC mass, m_0 (static mass with no movement), whose static force is $F_0 = m_0 g$, where g is the local gravity constant. In a dynamic situation (user jumping on the scale), a different, virtual mass, M_V, is measured in real-time. The total force F_T sensed is the rest-mass force F_0, plus the dynamic acceleration-causing force, F :

$$F = F_T - F_0 = (M_V - m_0) \cdot g \tag{1}$$

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Fig. 3. GCS300 gas cylinder load cell, providing a continuous-time signal from standing, jumping, and a variety of motions. The purpose of the gas is to distribute pressure for equal sensitivity at all points on the top surface. This device which ordinarily measures static weight, we repurpose to measure dynamic motion, and we convert the signal into *actergy* (time-integrated energy).

However, the acceleration from force F depends on the rest-mass, not the virtual mass: $F = m_0 a$, where a is acceleration of the body. We then examine the work W done along a progression of time. While constant-force work is $W = Fx$ (for displacement x), dynamic work over a slice of displacement δx is:

$$\delta W = F(t)\delta x = F(t)\delta t \cdot \frac{dx}{dt} \tag{2}$$

Integrating:

$$W = \int F(t)v(t)dt = \int F(t) \left[\int a(\tau)\ d\tau \right] dt \tag{3}$$

or,

$$W = \int F(t) \left[\int \frac{F(\tau)}{m_0} d\tau \right] dt \tag{4}$$

Therefore, the actergy is:

$$\aleph = \int W dt = g^2 \iint (M_V(t) - m_0) \left[\int \left(\frac{M_V(\tau)}{m_0} - 1 \right) d\tau \right] dt^2 \tag{5}$$

This simple analysis provided the basis for signal-processing of the electronic signal from the foot pad scale.

One future extension of this analysis is to correct for the fact that the accelerated mass (above the legs) is slightly smaller than the total body mass, for a brief moment when the legs are pushing the body upward but not yet themselves lifting off the ground. An anatomical analysis would lead to a slightly modified effective m_0 for that brief moment in time.

III. Further work: Integral-Kinematics to model physical processes

Absement, the time-integral of displacement, was introduced in [4] as a new fundamental kinematic quantity, creating a new unexplored part of the sequence of: displacement, velocity, acceleration, jerk, jounce, ... The derivatives of displacement are well-established, but the integrals of displacement have not been fully studied as fundamental kinematic quantities. Absement also has applications in electromagnetics and circuit theory [5], and, like actergy, may also provide a more detailed metric for kinetic performance than reflex time-delay. Fig. 1 shows higher-order integrals of displacement.

Aircraft dynamics is a further potential application of integral-kinematics, through a mathematical model for computer-efficient flight-control systems, as well as for physics education and gaming. We propose that absement-quantities can relate response of an aircraft to the pilot's control yoke motion, as follows. First, we define the following:

c_z control yoke displacement in the vertical elev. direction
θ_E elevator control surface angle
T_P torque in the pitch axis, caused by elevator
θ_P pitch angle, and ω_P angular velocity
I_P aircraft's moment of inertia in the pitch-axis.
Z altitude; β_{KA} kinetic-altitude conversion rate pitch factor

To a first-order approximation, the following relationships hold, and build up to a new conclusion using absement:

$$\theta_E \propto c_z \quad \text{for a displacement-sensitive yoke} \tag{6}$$

$$T_P \propto \theta_E \quad \text{for small } \theta_E,\ \theta_P, \text{ controlled airspeed} \tag{7}$$

$$\theta_P(t) \simeq \frac{1}{I_P} \iint_{t_0}^{t} T_P(\tau) d\tau^2 + \theta_P(t_0) \quad \text{under conditions}^1 \tag{8}$$

$$Z(t) \simeq \int_{t_0}^{t} \beta_{KA} \theta_P(\tau) d\tau + Z(t_0) \quad \text{under conditions}^2 \tag{9}$$

Therefore, to a first-order approximation for this aircraft type, the change in altitude is proportional to the third-order integral of the displacement of the control yoke:

$$\Delta\text{Altitude} \propto \Delta\textit{Abseleration} \text{ of the control yoke} \tag{10}$$

Abseleration is illustrated in Fig. 1. Even though pitch is an angle and not a linear measurement (*i.e.* absity-analogue of yoke position), the final result of altitude returns to a linear measurement (*i.e.* abseleration of yoke position).

Applications may include: reducing the computing load of flight-control computers; flight simulation and gaming; and education (teaching Newton's laws with reference to aircraft motion, in a simplified first-order model).

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[1] Assuming a large aircraft whose momentum dominates over pitch restorative forces and turbulence.

[2] Assuming engines set on auto-throttle to hold airspeed constant; this models conversion of forward-moving kinetic energy to vertical potential energy, according to a rate in proportion to pitch, for small pitch angles; this model requires symmetry for the case of zero rolling and yawing motion.

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